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Phil. Trans. R. Soc. Lond. A 1984 **313**, 19-25
doi: 10.1098/rsta.1984.0079

Phil. Trans. R. Soc. Lond. A **313**, 19–25 (1984)

Printed in Great Britain

The interaction of rotation and magnetic field in the Solar System

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During the early phases of star formation, torsional Alfvén waves propagating along the locally distorted galactic magnetic field can transport away the bulk of the initial angular momentum of a condensation, so enabling it to contract to solar nebula dimensions. Enough primeval magnetic flux may be retained for magnetic redistribution of angular momentum to continue until the proto-Sun reaches the pre-Main Sequence Hayashi phase. A rotating star with a convective envelope has a dynamo-maintained field which brakes the star through coupling to the stellar wind. Observational evidence from young star clusters and from T Tauri stars suggests that the zero-age Main Sequence Sun had about ten times its present angular momentum. The same braking process, scaled up because of the much more powerful T Tauri winds, can explain why the zero-age Sun had lost at least nine-tenths of the centrifugal upper limit, and is more acceptable than the suggestion that the 'missing' solar angular momentum has been magnetically fed into the planetary system.

INTRODUCTION

In his classical text *Astronomy and cosmogony*, when discussing the origin of the Solar System, Sir James Jeans argued against the fission theory because 'if one put all the angular momentum of the Solar System into the Sun (modelled as a liquid), the Sun would rotate 28 times as fast as it does now, but would still be far below centrifugal break-up'. Likewise he stated his objection to a Laplace-type nebular model: 'If the Sun once assumed the lenticular shape necessary for shedding of matter by rotation, it is difficult to see how it could ever abandon it and become as spherical as it is now.' Jeans was of course writing long before the introduction of electromagnetism into cosmical gas dynamics. His arguments assume implicitly that the present solar angular momentum – two orders of magnitude less than the centrifugal upper limit – has not sensibly altered since the Solar System was formed. In fact, satellite observations confirm that the Sun's rotation is being steadily braked by the magnetically controlled solar wind. It can be shown that the rate of loss of angular momentum is equivalent to that carried by the outflowing gas, supposed kept co-rotating with the Sun out to the surface S_A where the wind speed reaches the local Alfvén speed. It is difficult to construct a detailed model of the magnetic and velocity fields, and in particular to predict the dependence of the braking rate on the instantaneous angular velocity Ω. However, observations of line-widths in the spectra of G-type stars in the young Hyades and Pleiades clusters enable a tentative calibration to be made (Kraft 1967; Soderblom 1983). Extrapolation to the zero-age Main Sequence suggests that the Sun began its hydrogen-burning phase with $\Omega \approx 10$ times its present value.

Further evidence comes from observations of the T Tauri stars, plausibly identified with pre-Main Sequence stars with masses of solar order, and showing violent mechanical activity (Herbig 1983). The latest measurements (Vogel & Kuhi 1981) show them to be slowly rotating, with angular momenta close to that of the zero-age Sun (account being taken of the larger radii of stars that have not yet reached the Main Sequence). The faster rotators among them appear to be generally of higher mass, as is the case on the Main Sequence.

The observations thus appear to assign to the Sun and other low-mass stars a zero-age rotation that is one order of magnitude less than the centrifugal limit. The dynamo-built magnetic fields of these late-type stars with extensive outer convective zones are clearly vital to the rotational evolution on the Main Sequence; we now ask when magnetic fields – 'fossil' or dynamo-built – are significant during the earlier epochs. How much appeal plausibly may be made to braking by the galactic magnetic field during the earlier diffuse phases? Is a scaled-up version of the magnetically controlled wind during the pre-Main Sequence phase a likely explanation of the missing second order of magnitude of angular momentum? Is there an epoch during the later phases of star formation in which magnetic effects are unimportant? And how likely is the suggestion that much of the missing solar angular momentum has been magnetically fed into the surrounding proto-planetary material?

THE EARLY PHASES OF STAR FORMATION

With the currently accepted value for the large-scale galactic magnetic field ($B \approx 3 \times 10^{-10}$ T, associated with a proton number density of *ca.* 1/cm^3), it is unrealistic to ignore magnetic forces during the early phases of star formation. If a self-gravitating mass M – whether a cloud, or a subcondensation such as a cloud-core – is threaded by magnetic flux F, then there exists a critical mass $M_c \approx kF/\pi\sqrt{G}$ with the numerical factor k near unity (its exact value depending on the shape of the condensation and the details of the mass-flux distribution). If $M < M_c$, then a cool body settles into a state of magneto-gravitational equilibrium with a trans-field radius (Mestel 1965; Strittmatter 1966)

$$\bar{R} \approx R_0[1-(M/M_c)^2], \tag{1}$$

and the frozen-in mean field $\bar{B}$ within the mass related to the background external field B_0 by $\bar{B} = B_0(R_0/\bar{R})^2$. (Applied to the cloud as a whole, B_0 could be the galactic field, approximately 3×10^{-10} T; to a subcondensation, the mean field within the cloud.) Without the magnetic field a cool condensation would contract, conserving the angular momentum it would acquire naturally from the galactic rotation and the galactic turbulence; and with realistic numbers inserted, it is easy to convince oneself that the growing centrifugal forces of spin would halt the contraction in two dimensions at quite moderate densities ('the angular momentum problem'). The galactic field not only halts the contraction of a subcritical mass at the radius $\bar{R}$, but should also enforce near *co-rotation* with the background.

Within a molecular cloud (the most likely locale of star formation), an initially subcritical mass can become supercritical through slow but significant flux-leakage (Mestel & Spitzer 1956). The forces exerted by the locally distorted magnetic field cause the electrons and ions to drift relative to the neutral bulk, at a rate given by a balance between magnetic force and the ion-neutral particle friction; and in a molecular cloud the ion-neutral ratio becomes small enough for this drift of charged particles together with the inductively coupled magnetic field to reduce F significantly within a cosmical timescale. As F (and so also M_c) slowly declines, the mass contracts, but the magnetic stresses can still enforce co-rotation; and the closer M_c is to M, the larger the proportion of the initial angular momentum transferred to the background. The most optimistic estimates can yield a specific angular momentum of the same order as that in an inferred solar nebula: e.g. if an angular velocity of around 10^{-15} s^{-1} (of the order of the galactic rotation) is maintained by a mass of a few times the solar mass up to densities of approximately 10^9 particles cm^{-3}. Some caution is probably in order here: the efficiency of angular momentum transport by Alfvén waves will be markedly reduced once the condensation field lines begin to detach from the background. However, the following tentative conclusions are reasonable:

(1) the galactic magnetic field can remove the bulk of the angular momentum of an initially subcritical mass, while leaving enough to account for binary stars and the Solar System (Mestel 1965; Mouschovias 1977, 1983; Mestel & Paris 1979);

(2) the steady adjustment to slow flux-leakage of the co-rotating, subcritical mass yields a mass-angular momentum distribution qualitatively similar to that in the Solar System;

(3) once flux leakage has yielded $M > M_c(F)$, the slowly rotating mass will collapse with near conservation of its remaining angular momentum, but the consequent spin-up will not yield centrifugal forces again comparable with gravity until much higher densities are reached;

(4) the flux-leakage time may still be longer than the free-fall time even after $M > M_c(F)$. If so, the collapsing mass will begin by dragging its remaining flux with it (Black & Scott 1982), so that the field may remain dynamically significant ($M > M_c(F)$ but not $M \gg M_c(F)$).

The intermediate phases

The last point suggests that once the centrifugal forces have grown large enough to slow up the collapse, contraction of at least the inner regions may nevertheless continue through further magnetic redistribution of angular momentum. This would be particularly important if (as is likely) the collapse is not halted until densities where the gas is highly opaque, so that contraction of a non-rotating body would be determined by heat loss rather than by gravity, thus giving a longer timescale for the magnetic stresses to act. At these densities most of the field lines will have detached from the background field, and more angular momentum will be lost from regions linked to the surroundings by field lines furthest from the O-type neutral points. If the magnetic and rotation axes are not inclined at a large angle, this will favour the central parts, so increasing still further the outward angular momentum gradient. If the rotation axis is not strictly parallel to the magnetic axis, the mutual interaction of the mass and the surroundings yields also a precessional torque which normally tends to align the two axes (Paris 1971). This will again act preferentially on the gas near the centre, and so could cause a net inclination between the angular momentum vector of the gas destined to form the proto-Sun, and the surrounding disc material. Equatorial settling of dust in the outermost parts of the disc, followed by gravitational instability in the dust layer (Goldreich & Ward 1973) could be the origin of the Oort cometary cloud (Biermann & Michel 1978).

At this point, however, there arises the crucial question of the stability of detached magnetic fields. It is known (Wright 1973; Markey & Tayler 1973, 1974; Tayler 1973) that even weak stellar fields of simple topology are subject near their O-type neutral points to the analogues of familiar laboratory magnetohydrodynamic instabilities. The nonlinear development of the unstable modes is unknown, and it is possible that in a differentially rotating body the field may acquire (through relaxation of strict flux-freezing) a structure with mutually linking poloidal and toroidal fields that is *prima facie* dynamically stable. If, however, no such stable structure exists when magnetic and gravitational energies are comparable, then presumably substantial flux destruction occurs. A comparatively weak field that is dynamically stable can still be cosmogonically significant, because of the more relaxed (thermal) timescale. Slow flux-loss probably continues through secular instabilities, dependent on heat exchange and finite resistivity. It is thus clearly very difficult to say how important magnetic effects are in this phase. At one extreme, we have the case when so much angular momentum is lost during the earliest diffuse phases that the proto-Sun can reach the top of the Hayashi track before the centrifugal force approaches gravity. If the bulk of the primeval flux is rapidly expelled by the turbulence, the subsequent evolution of the rotation would be via the dynamo-built magnetic field (see later). In the perhaps more likely event that the diffuse phase braking is rather less efficient, then the uncertainties in our understanding of stability force us to leave open whether there are significant further effects of the primeval flux on the angular momentum. It could be that this phase of the evolution of an opaque, rapidly rotating mass is purely gas-dynamical until the Hayashi track is reached and dynamo action starts.

There is no question but that the bulk of the primeval flux must be lost at *some* epoch. (Even in the strongest observed magnetic star with a surface field of around 3.5 T, no plausible inward extrapolation of the field yields a total flux that is more than a small fraction of the upper limit corresponding to $M = M_c(F)$.) The T Tauri stars are known to emit a remarkable fraction – 10 % or so – of their luminosity as mechanical energy, and the suggestion has been made that the cause of this phenomenon is the availability for dissipation of a substantial excess primeval magnetic flux (implying that losses, e.g. by instabilities in earlier phases, have still left a non-trivial reservoir). It is tempting to assume this and ask whether there could be an associated significant loss of angular momentum. Thus we write a phenomenological equation for the supply of energy to the T Tauri wind, supposing that a magnetic energy

$$\mathcal{M} \approx \tfrac{1}{3}\bar{B}^2 R^3 = \eta G M^2/R \tag{2}$$

is being destroyed in a characteristic time τ_B, with a fraction ϵ being available as wind kinetic energy:

$$\tfrac{1}{2}|\dot{M}|\, v^2 = \epsilon \mathcal{M}/\tau_B\,. \tag{3}$$

Here $\bar{B}$ is the mean internal field strength, $\mathcal{M}$ is a fraction η of the gravitational energy, and $|\dot{M}|$ is the mass loss rate. As noted earlier, the angular momentum loss rate is equivalent to exact co-rotation out to the Alfvénic surface of radius r_A, so that the braking of a star with moment of inertia kMR^2 ($k \approx \frac{1}{10}$) is described by

$$-\frac{\mathrm{d}}{\mathrm{d}t}(kMR^2\Omega) = \tfrac{2}{3}|\dot{M}|\Omega r_A^2. \tag{4}$$

To avoid exaggerating the braking, the external field is approximated as dipolar rather than radial. For consistency, (4) must predict a braking time τ_Ω that is no longer than τ_B. Combination of (2), (3) and (4) with the equation of continuity then requires

$$\tau_B > \tau_{m1}\left[\left(\frac{\epsilon}{\eta}\right)^{\frac{1}{5}}\left(\frac{k\bar{B}}{B_s}\right)^{\frac{4}{5}}\left(\frac{\tau_{ff}}{\tau_{m1}}\right)^{\frac{2}{5}}\right], \tag{5}$$

where $\tau_{m1} = M/|\dot{M}|$ is the characteristic mass-loss time, τ_{ff} is the free-fall time $(GM/R^3)^{-\frac{1}{2}}$, and B_s is the surface field strength. With $\tau_{m1} \approx 10^8$–10^9 years (a conservative figure for T Tauri stars), and a reasonable estimate for $\bar{B}/B_s$, (5) then requires that τ_B exceed 10^5 years: a shorter time of flux-loss would make the gas reach the Alfvén speed too quickly and so yield too small

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an angular momentum loss rate. It is convenient that (5) depends so weakly on the unknown parameters ϵ and η. To be *prima facie* acceptable, the proposal must not prove too much. The more massive stars must be left with a higher specific angular momentum. Further, an exponential law of angular momentum decay could be embarrassing: what is needed is a process that cuts off near the T Tauri, zero-age solar angular momentum (e.g. an acceleration of flux-loss at slower rotations). It could be that the proposal raises more astronomical problems than it solves; nevertheless, the problem of explaining the dynamics of the T Tauri phenomenon remains. Some help may come from projected optical Zeeman effect measurements of T Tauri spectral lines.

Pre-Main Sequence dynamo-built fields

Dynamo-maintained fields are expected at the surfaces of rotating stars with strong outer convective zones. The fields with the strongest dependence on rotation probably result from the '$\alpha\Omega$' dynamos, described by the equations

$$\partial \boldsymbol{B}_{\mathrm{t}}/\partial t = \tilde{\omega}(\boldsymbol{B}_{\mathrm{p}}\cdot\nabla)\,\Omega, \tag{6}$$

$$\partial \boldsymbol{B}_{\mathrm{p}}/\partial t = \nabla\times(\alpha\boldsymbol{B}_{\mathrm{t}}), \tag{7}$$

where dissipative terms have been dropped. Equation (6) describes the familiar twisting of a poloidal field $\boldsymbol{B}_{\mathrm{p}}$ by a non-uniform rotation field Ω. In equation (7) there is an effective electromotive force $\alpha\boldsymbol{B}_{\mathrm{t}}$ deriving from the effect of Coriolis force on convective motions, as pointed out in a seminal paper by Parker (1955). Thus Ω is crucial to both parts of the cycle $\boldsymbol{B}_{\mathrm{p}}\rightarrow\boldsymbol{B}_{\mathrm{t}}\rightarrow\boldsymbol{B}_{\mathrm{p}}$. To understand magnetic braking of late-type stars, we need to know how the strength of the dynamo-maintained field depends on spectral type and on Ω. One can try to gain insight by again appealing to observations of G-type stars. The simplest braking model (Weber & Davis 1967) assumes a thermally driven, spherically symmetric wind with an associated *radial* external $\boldsymbol{B}$-field. Equation (4) then converts to

$$-\frac{\mathrm{d}}{\mathrm{d}t}(kMR^2\Omega) = \tfrac{2}{3}\Omega(B_{\mathrm{s}}^2R^4/v_{\mathrm{A}}), \tag{8}$$

where v_{A} is the wind speed at S_{A}. Note how the mass-loss rate has disappeared: in this geometry, an increase in density is exactly compensated by a decrease in r_{A}. If one then supposes that $B_{\mathrm{s}} \propto \Omega$ (Durney & Stenflo 1972), and with the trans-sonic speed v_{A} nearly constant, equation (8) predicts $\Omega \propto t^{-\frac{1}{2}}$, which was thought to fit the observed values for the Sun and younger G-type stars (Skumanich 1972). The agreement was in fact slightly embarrassing, since it is questionable whether the field within the Alfvénic surface S_{A} should be radial: with the star slowly rotating and the wind thermally driven, one expects a quasi-dipolar field with an extensive dead zone (Mestel 1967, 1968), a picture qualitatively vindicated by subsequent X-ray observations of the solar corona. At higher rotation rates, however, the centrifugal forces of approximate co-rotation take over from the thermal pressure well within S_{A}, and consequently one may expect more of the field lines emanating from the star to be pulled out to form a quasi-radial wind-zone beyond S_{A}. If $B_{\mathrm{s}} \propto \Omega$ is still provisionally adopted, then there results a stronger dependence on $1/t$, as seems to be indicated by revised observational results (N. O. Weiss, personal communication). With a radial field assumed, and with $v_{\mathrm{A}} \approx \Omega r_{\mathrm{A}}$ (a 'centrifugal wind'), one finds $\Omega \propto 1/t^{\frac{3}{2}}$, and the time τ_{Ω} for the star to lose 90% of its initial angular momentum $\propto$ (mass-loss time)$^{\frac{1}{3}}$; the inferred T Tauri mass-loss time of 10^8–10^9 years then yields $\tau_{\Omega} \approx 2.5\times10^7$ years, which is of the right order for pre-Main Sequence evolution.

This time estimate depends more critically on the shorter mass-loss time and on the adoption of a quasi-radial field than on the $B_s \propto \Omega$ hypothesis, but one still wants to know the $B_s(\Omega)$ relation for both moderately slow and fast rotators. Equations (6) and (7) already involve implicitly some dynamics through the dependence of the $\alpha \boldsymbol{B}_t$ term on the Coriolis force. The limitation to the growth in the field must come from some back-reaction of the magnetic force on the velocity field. Magnetic buoyancy – the spontaneous tendency of horizontal flux-tubes to float to the surface – is an efficient means of balancing the field amplification, which from (6) and (7) has a characteristic time of *ca.* $(1/\alpha\,|\nabla\Omega|)^{\frac{1}{2}}$. If the 'helicity' function α is proportional to Ω over the relevant parameter range, if $|\nabla\Omega| \approx \Omega/R$, and with the rise time of buoyant flux-tubes proportional to the inverse of the Alfvén speed, then indeed $\bar{B} \propto \Omega$ (Durney & Robinson 1982). Knobloch *et al.* (1981) also appeal to magnetic buoyancy, but point to a possible change from an oscillatory to a d.c. dynamo at higher Ω, which may increase B_s and so also the braking rate.

Progress on this dynamical dynamo problem is likely to be slow. There is probably more scope for improving our understanding of the magnetosphere, e.g. in elucidating the variation of the wind zone with increasing Ω. The higher mass-loss rates of the T Tauri stars should also be predicted from theory rather than inferred from observation. What determines the mass-loss rate is the point in the stellar atmosphere where the wind speed becomes sonic. The most realistic description of a T Tauri wind may be 'magneto-centrifugal', with the effective pressure of random, small-wavelength Alfvén waves dominating over thermal pressure near the star (see, for example, Hartmann *et al.* 1982). But in spite of all the uncertainties, one feels that there are no overwhelming arguments against the hypothesis that if the Sun and other solar-type stars begin the descent of the Hayashi track with an angular momentum near to the centrifugal limit, then they will lose one order of magnitude in the pre-Main Sequence phase through the enhanced stellar wind coupled to a dynamo-built field.

The observational and theoretical evidence in favour of braking by winds argues against the suggestion that the missing solar angular momentum has gone into the planets. There are no intrinsic difficulties in endowing proto-planetary material with its angular momentum: on the contrary, one has to look for processes that ensure that the expected original angular momentum does not prevent stellar and planetary systems from forming. Even with the most efficient likely combination of magnetic braking and flux diffusion in the earliest phases, there is every reason for expecting a proto-solar nebula to have more than enough high angular momentum material to form the planets. The simplest hypothesis is that the missing solar angular momentum is now irrevocably mixed with that of the local interstellar kinetic field.

Contemporary Solar System magnetic fields

There remain gaps in our understanding of solar and stellar dynamos, in particular the precise explanation of the basic non-uniform rotation, the reason for the excitation of a particular dynamo mode, and the cause of the inevitable dynamical limitation to growth. The physical problems do not, however, include that of energy supply: a late-type star has available for tapping a huge reservoir of turbulent energy in its convective envelope. Planetary magnetic fields are also dynamo-maintained, and simple arguments based on equations (6) and (7) with the Ohmic terms restored suggest that all planets and satellites rotate rapidly enough to have active dynamos, provided they have convective cores (Stevenson 1983). The energy supply is now crucial. Compositional convection due to gravitational settling – the steady freeze-out of a core – appears to be adequate to maintain the fields of the Earth and of Mercury; the absence of strong fields in Venus, Mars and the Moon may very well be due either to the virtual absence of a growing inner core, or to an insufficient associated energy release. Thermal or combined thermal-compositional convection supplies the energy for the fields of the giant planets.

Finally, a word on the internal solar rotation: over the long timescales available, even a very weak magnetic field beneath the solar convection zone will interact with a shear (a torsional Alfvén wave along a field as low as 10^{-10} T would travel through the core in a solar lifetime). Incontrovertible evidence, e.g. from solar seismology of an angular velocity gradient (cf. Gough 1984, this symposium) would set problems not only for the hypothesis of a relic fossil field in the solar core but also for the solar dynamo, which several workers now think of as being located in the convective overshoot layer separating the fully convective envelope from the stable core.

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